

Selected topics in graph theory

Homework #3

Date: May 18, 2023

Due: May 29, 2023.

(Solutions must be submitted electronically. I recommend pdf compiled from latex.

Send it to lovasz@caesar.elte.hu.)

1.1 PROBLEM. Prove that for any three simple graphs F, G, H , we have

$$t_{\text{inj}}(F, H) \geq t_{\text{inj}}(F, G)t_{\text{inj}}(G, H).$$

1.2 PROBLEM. Let $K_{n,n}$ denote the complete bipartite graph with n nodes in each color class. Prove that for every graphon W ,

$$t(K_{n,n}, W) \geq t(K_2, W)^{n^2}.$$

1.3 PROBLEM. We call a graphon W bipartite, if there is a partition $[0, 1] = V_1 \cup V_2$ such that $W(x_1, x_2) = 0$ for almost all $(x_1, x_2) \in V_1 \times V_1$ as well as for almost all $(x_1, x_2) \in V_2 \times V_2$. Prove that a graphon is bipartite if and only if $t(C_{2k+1}, W) = 0$ for all $k \geq 1$. (Here C_n denotes the cycle on n nodes.)

1.4 PROBLEM. Prove that for every graphon W ,

$$t(K_3, W) + t(K_3, 1 - W) \geq \frac{1}{4}.$$

1.5 PROBLEM. Let F and G be two simple graphs, and let W be a graphon such that $t(F, G) > 0$ and $t(G, W) > 0$. Prove that $t(F, W) > 0$. [Hint: Use the Lebesgue Density Theorem.]